Nonstationarity Measure

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Outline

- Background and motivation
- Stationary vs nonstationary
- Nonstationarity measure
- Computation of Nonstationarity measure
- Conclusion

Complex system

- Complex system is “non”

    nonlinear

    nonstationary

    nonequilibrium

    … …

Stationary vs nonstationary

Stationary: statistical properties is invariant

- strict stationary:all of the finite dimensional joint distributions are time-invariant
- weak stationary:mean and covariance are time-invariant

Nonstationary:

- trends(seasonal, scaling,cyclic,growth…)

Yes or No

Time and uncertainty are two most important factors when an economic agent makes a decision.

Economics and finance study the behaviors of economic agents in allocating their resources among different alternatives and over different time periods in an uncertain environment.

Stationary assumption

A and B are stationary process, how can one say process A is more stationary than process B?

- IID
- Zero serial dependence
- Short memory
- Long memory

A and B are nonstationary process, how can one say process A is more nonstationary than process B?

Challenge

In particular, for same time series or data stream, different detrend method reduce different trends, which one is more suitable?

Consider the nonstationarity measure of residue series.

General time series analysis

Given some observations on $\{X_t\}$, find a function $h(.)$ such that $\{\varepsilon_t\}$ is an i.i.d. sequence, where

$$h(X_t, X_{t-1}, X_{t-2}, \ldots) = \varepsilon_t.$$

The function $h(.)$ captures the dynamic structure of $\{X_t\}$.

Priestly, M.B.(1981), Time Series and Spectral Analysis.

Satble set----coarse grain

- Time series(data stream)

$$X = \{x_k, k = 1, 2, \dots\}$$

- Frequency sequence of A

$$f_n(A) = \frac{\#\{i : x_i \ lies \ in \ A, i \le n\}}{n}$$

- Stable set: Frequency sequence converges

$$f_n(A) \to f(A) \quad as \ n \to \infty.$$

Stable set admits reliable probability

- The phase space S that the data stream X lies is always stable.
- If the subset A is stable with respect to X, then the complementary set S\A is also stable with respect to X.
- Suppose two disjoint sets A and B are stable with respect to X, then the union of A and B is also stable with respect to X.

For given data stream X, we can assign the a probability on each stable set of it, and such a probability is time-invariant.

$$f_n(A) = \frac{\#\{i : x_i \text{ lies in } A, i \leq n\}}{n}$$

$$f_n(A) \to f(A) \quad \text{as } n \to \infty.$$

Information structure of data stream X

- Partition of the phase space S
- Stable partition of X is called a information structure of X
- Stationary process admits finest information structure----each subset of S is stable by ergodic theorem of stationary process
- Nonstationary process----some subsets of S may not stable

General ideas

A data stream X is likely more stationary than Y if X admits more stable sets than Y.

Y is more nonstationary than X because Y admits less stable sets than X.

More or less

mathematically?

Nonstationarity measure

- Information structure

$$IS = \begin{pmatrix} A_1 & A_2 & A_3 & \cdots & A_k \\ p_1 & p_2 & p_3 & \cdots & p_k \end{pmatrix}$$

- Shannon entropy

$$H(IS) = -\sum p_i \log p_i .$$

- Nonstationary measure NS(X) of X is defined as the maximun Shanon entropy among all information structure of X.
- NS(X)>NS(Y) implies X is more stationary than Y, or Y is more nonstationary than X.

Why and How

- Shannon entropy is widely used in many fields….
- Shannon entropy can compare

  more or less in terms of

  finer or coarser of two partitions
- Shannon entropy captures probanilities in the information structure
- ……

Pratical issues:
finite length of samples

- How to calculate the nonstationarity of X?

  Obtain the finest information structure (stable partition) of X

- Approximation

finest---- among a pre-assumed partition

stable---- criterion to ensure convergence

Procedures: approximate computation of nonstationarity measure

Given a data stream X in phase space S

1, Partion S into finite (small) pieces

Example: S=[0, 1], partition S into 100 intervals with equal length

2, Select a criterion ----some kind of convergence rate. Example: A is stable if

$$| f_n(A) - f(A) | < C/n^r ,$$

$$f(A) \text{ is the average of last } 20\% \text{ of } f_n(A).$$

SP—finest stable partition among all partitions coarser than the pre-assumed partition

H(SP)—good approximate of NS(X)

3, Use the criterion, check if the single piece in the pre-assumed partition is stable or not, take the stable one into the stable partition SP.

4, For the remain pieces (each one is not stable), check if the union of two pieces is stable or not, take those stable one into the stable partition SP.

5, Continue work on the remain piecese, check if the union of three pieces is stable or not……

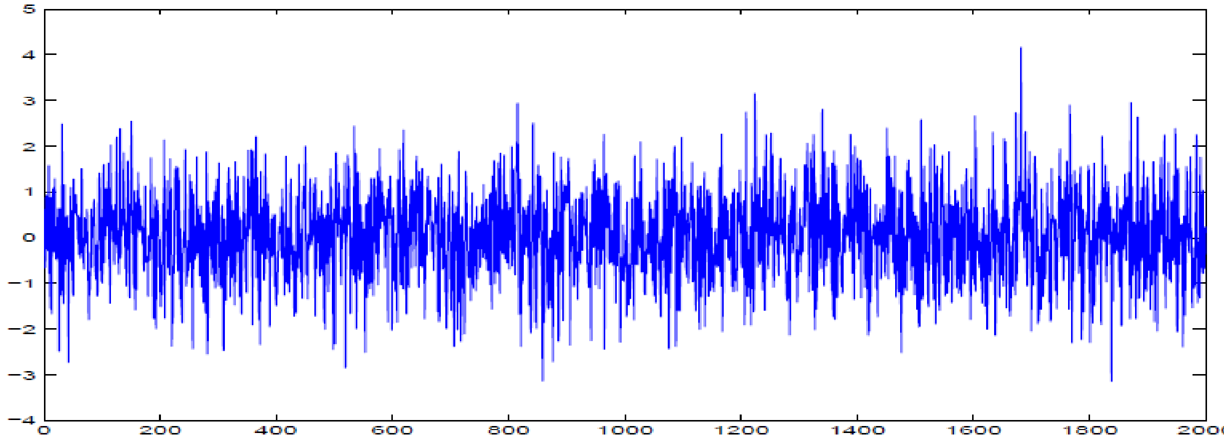
6, Stop when the weight of remain pieces is small.

Data set

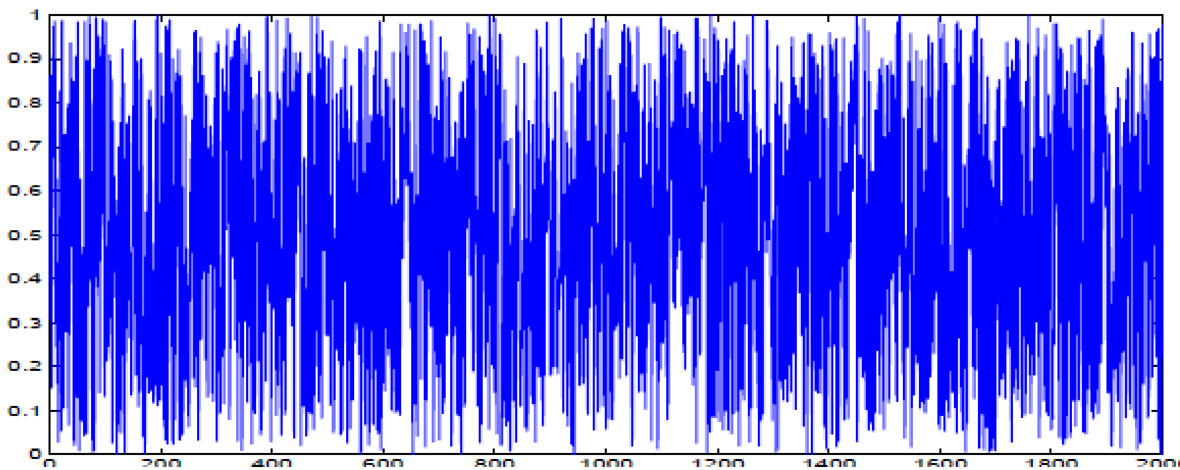
- IID N(0, 1): length 2000
- IID U(0, 1): length 2000
- Logistic map: length 2000 $x_t = 4x_{t-1}(1 - x_{t-1}).$

- FTSE 100: 2000.1.4----2007.12.31, length 2020
- NIKKEI 225: 2000.1.4----2007.12.28, length 1967
- S&P500: 2000.1.3----2007.12.31, length 2010
- SSE Composite index:2000.1.4----2007.12.31, length 2076.

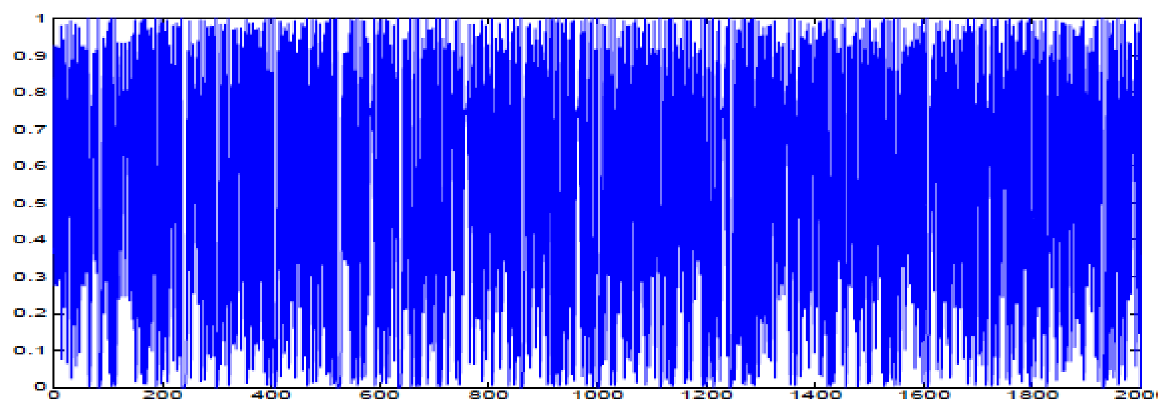
IID N(0,1)

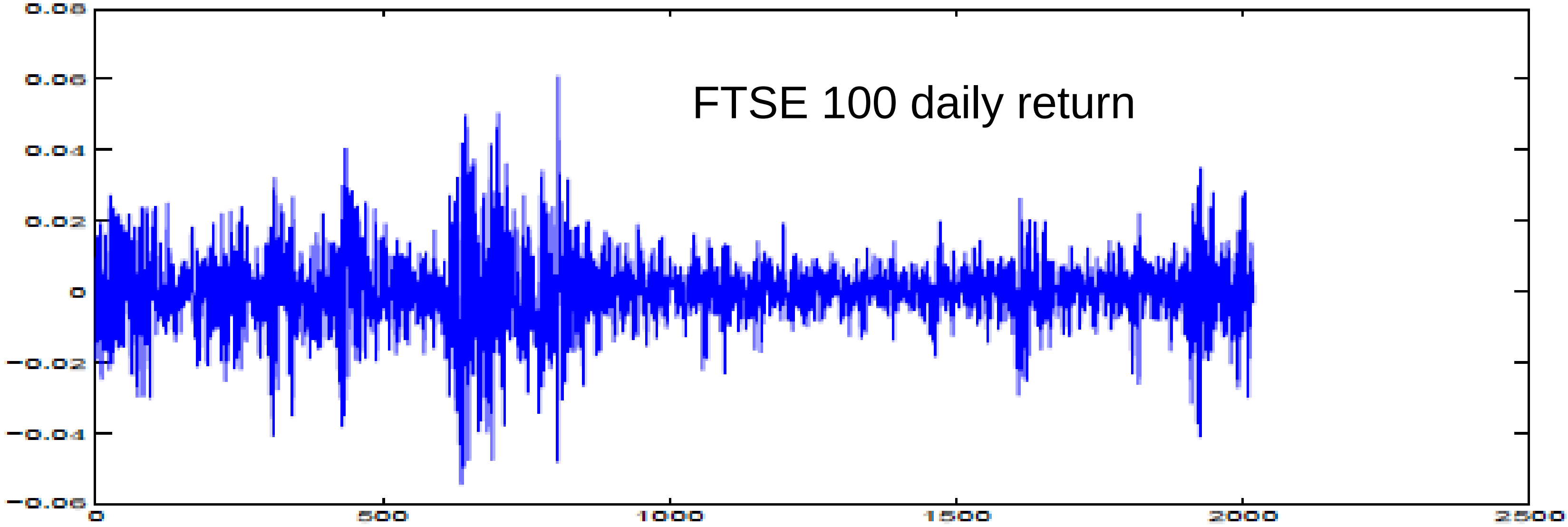


IID U(0, 1)

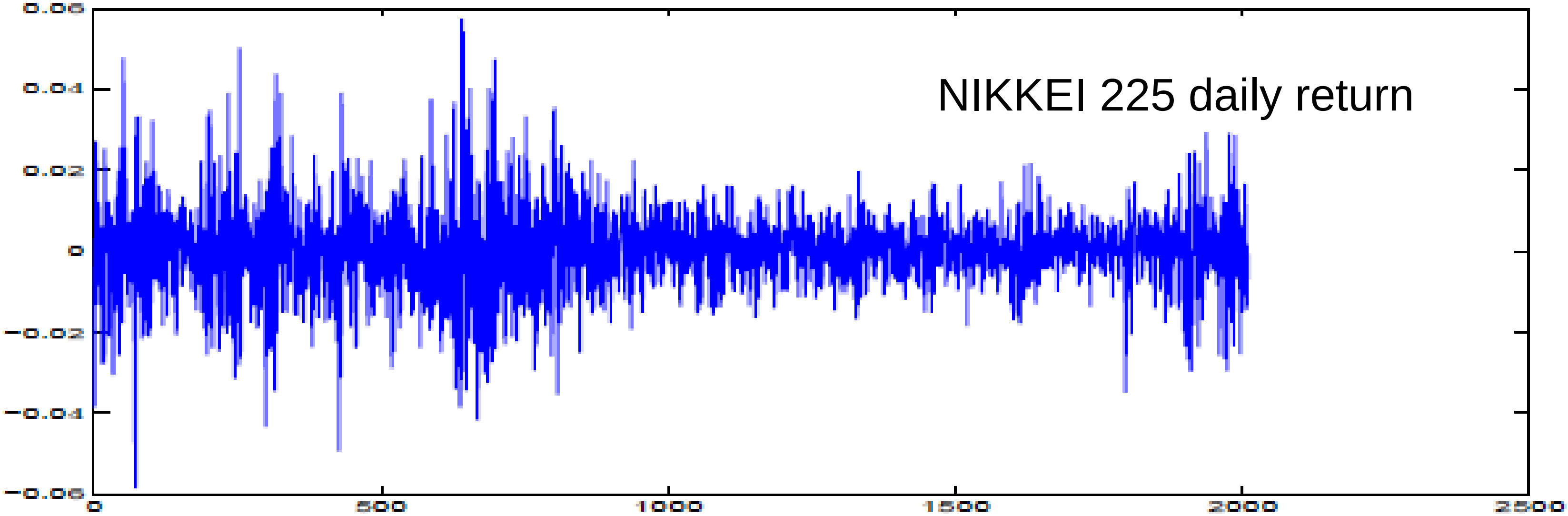


$$x_t = 4x_{t-1}(1 - x_{t-1})$$

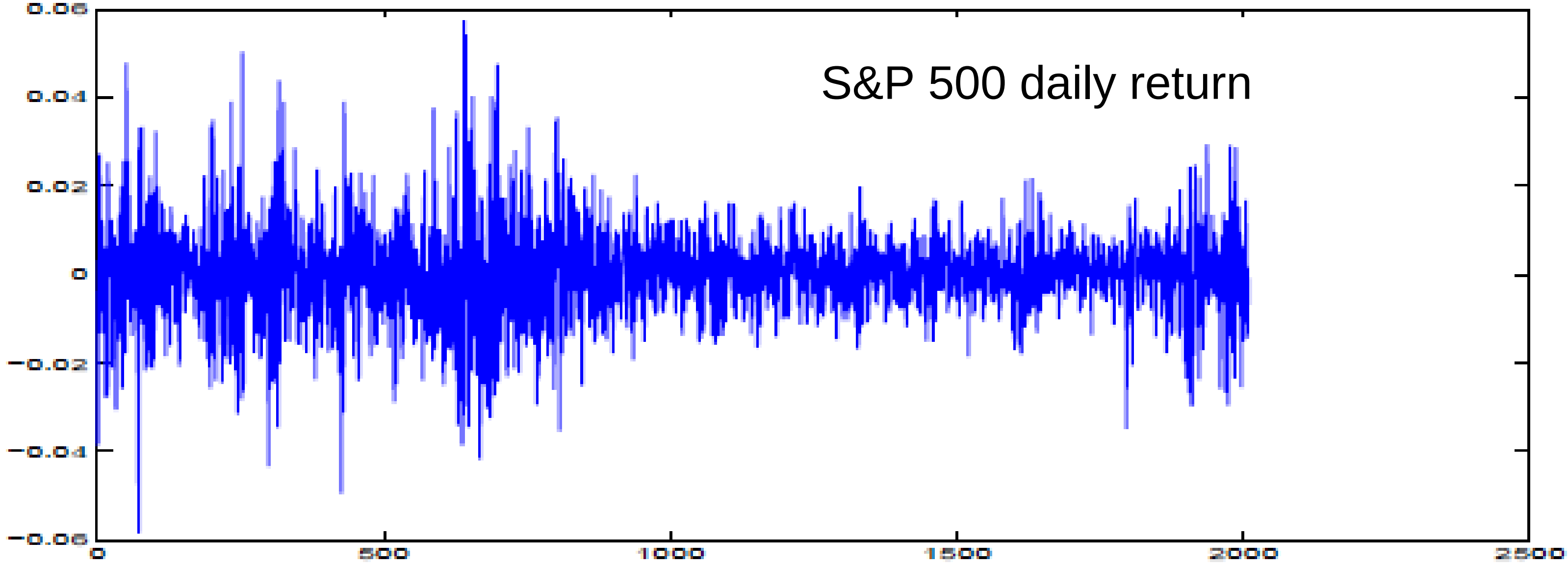


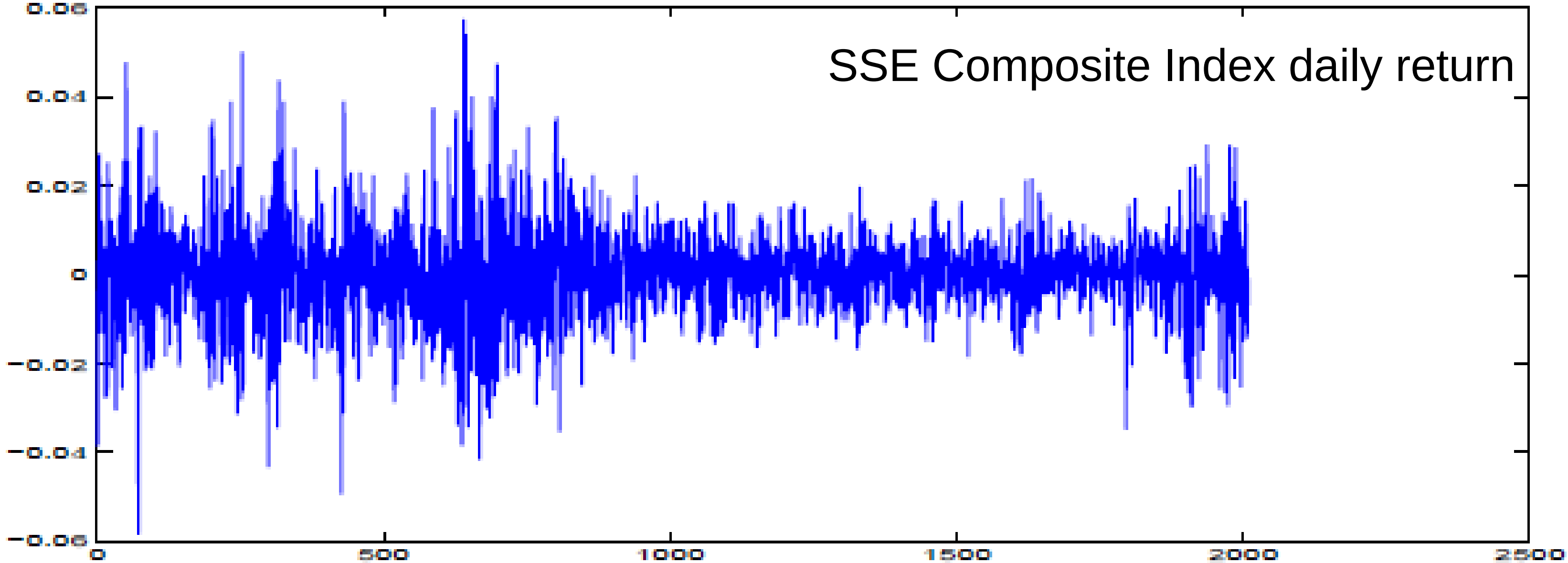


FTSE 100 daily return
0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
0
500
1000
1500
2000
2500



NIKKEI 225 daily return
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
0
500
1000
1500
2000
2500


S&P 500 daily return
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
0
500
1000
1500
2000
2500


SSE Composite Index daily return
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
0
500
1000
1500
2000
2500

- Partition each phase space into 50 equal pieces
- Convergence criterion:

$$Y = \{y_1, y_2, ..., y_T\} \ \ \textit{converegs to} \ y_T \ \textit{if}$$

$$\#\{y_i \in Y : \frac{|y_i - y_T|}{T - i} \frac{T}{y_T} \le \gamma, i = 1, 2, ..., T\} \ge \alpha T.$$

Paremeters: $\alpha = 0.9, \gamma = 0.1, 0.2, ..., 1.0.$

90% of points in a frequency sequence lie in a linear strip controlled by gamma around the limit.

Data	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ussian[a]	2.3488	3.5650	3.7768	3.8860	4.1495	4.2553	4.2737	4.4416	4.5606
iform[a]	0.8211	4.1421	4.5596	4.6819	4.8420	4.9251	5.0171	4.9802	4.9231
gistic[a]	1.8721	4.0068	4.3632	4.5966	4.7423	4.7713	4.9012	4.9314	4.9911

Data	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
FTSE 100[a]	2.1066	2.8647	2.8317	3.2178	3.4306	3.5420	3.6301	3.8069	3.7698
S&P 500 Index[a]	1.8299	2.6397	2.8180	2.9966	3.4156	3.5371	3.7178	3.8777	3.9269
NIKKEI 225[a]	2.1662	2.8458	3.2706	3.6163	3.7211	3.8862	3.9573	3.9471	3.9452
E Composite Index[a]	1.0399	1.9708	2.6294	2.7553	2.6989	2.6630	2.8766	2.8736	2.9607

Results

Data	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	mode
Gaussian	1	3	3	3	3	3	3	3	3	3	3
Uniform	7	1	1	1	1	1	1	1	2	2	1
Logistic	4	2	2	2	2	2	2	2	1	1	2
FTSE 100	3	4	5	5	5	5	6	6	6	6	5
NIKKEI 225	2	5	4	4	4	4	4	4	4	5	4
S&P 500 Index	5	6	6	6	6	6	5	5	5	4	6
SSE Composite Index	6	7	7	7	7	7	7	7	7	7	7

Conclusions

- It is necessary and possible to compare the (non)stationary levels of different data streams
- Use coarse grain and stablity of frequency sequence to obtain stable partition (information structure)
- Use Shannon entropy of the information structure to compare the finess of different information structures
- Effective approximate methods

Thank you

for your attention!

Comments or debates?

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