Measuring the Growth and the Structural Change of a Multi-Sector Economy

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Previous studies

Economic kinematics: *not found*

Structural change:

- *Moore, J.H. (1978):* A Measure of Structural Change in Output. *Review of Income and Wealth*, ***24*** *(1).*
- *Poirier, D.J. (1976)*: The Econometrics of Structural Change. *North Holland Publishing Company.*
- *Ploberger, W. & Kramer, W. (1986):* On Studentizing a Test for Structural Change. *Economics Letters*, ***20***, *341-4.*
- *Dufour, J-M. (1982):* Generalized Chow Tests for Structural Change: A Coordinate free Approach. *International Economic Review*, ***23*** *(3).*

Part 1: Analysis of economic production and its growth

Kinematics

”Kinematics is the study of the geometry of motion; it deals with the mathematical description of motion in terms of position, velocity, and acceleration. Kinematics serves as a prelude to dynamics, which studies force as the cause of changes in motion.”

Ohanian, H.C. Physics, 2nd Ed. (1989) p. 25.

Economic kinematics

- A change in production in an *n*-sector economy consists of *n* one-dimensional changes occurring simultaneously.
- We describe this process by a vector function illustrating the motion of an ideal particle with no size and no internal structure.
- This ideal particle represented by a point in the *n*-space of accumulated sectoral productions illustrates the position of the economy at a particular moment of time. Accumulated production values in the coordinate axes guarantee that the economy moves forward in the *n*-space with time, and it corresponds to (accumulated) lengths in coordinate axes in physics when modelling the motion of particles.
- Our data is current annual values and annual values at 2000 prices (= volumes) from 9 sectors in Finland at 1975-2002.

The data contains 9 sectors in Finland at 1975-2002:

1 = A (Agriculture, forestry and hunting) + B (Fishing),

2 = C (Mining and quarrying) + D (Manufacturing) +
E (Electricity, gas, and water supply),

3 = F (Construction),

4 = G (Trade, repair of motor vehicles and household goods)
+ H (Hotels and restaurants),

5 = I (Transport, storage and communication),

6 = J (Financial intermediation and insurance),

7 = K (Real estate and business activities),

8 = L (Administration, compuls. social secur.) + M(Education),

9 = N (Health and social work) + O (Other community,
social and personal services) + P (Household service activities)
- Financial intermediation services indirectly measured.

These sectors cover the whole Finnish production.

Accumulated values in primary (sector 1) and secondary production (sector 2), and services (sectors 3+...+9)

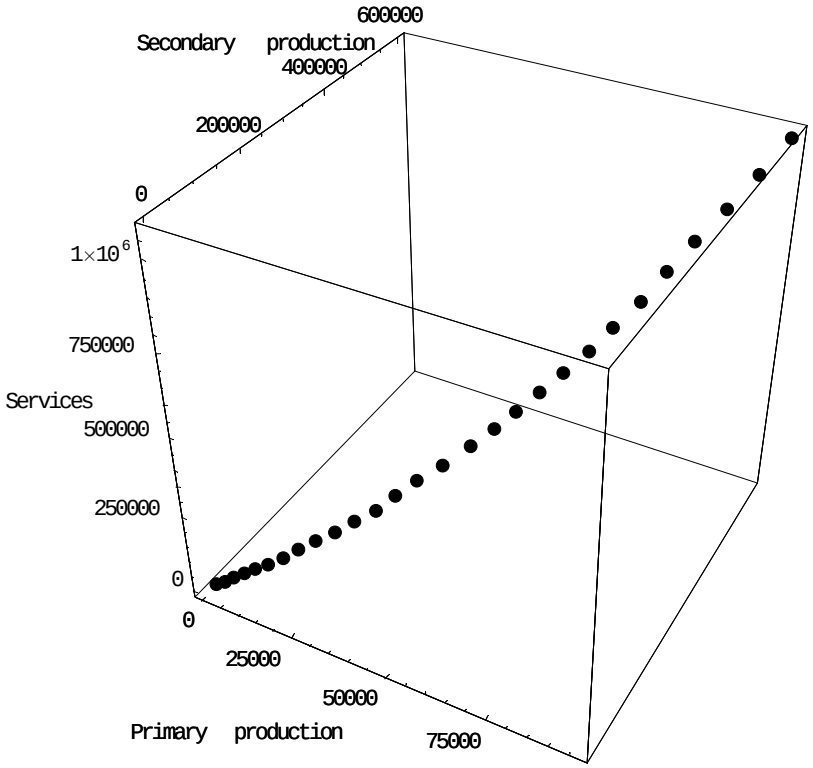


Figure 1. The three-dimensional accumulated production structure in Finland in value terms at 1975-2002

The coordinate system is the following of accumulated production values of *n* sectors:

$$X_1 = \int Q_1(s)ds, \ldots, X_n = \int Q_n(s)ds.$$

Qi(s) has unit *€/year* and thus ∫*Qi(s)ds* has unit € forall *i*. At time *t*, the position vector of the ”point economy” is:

$$\mathbf{X}(t) = \left(\int_0^t Q_1(s)ds, \ldots, \int_0^t Q_n(s)ds \right),$$

with the following velocity and acceleration vectors:

$$\mathbf{X}'(t) = \left(Q_1(t), \ldots, Q_n(t)\right),$$
$$\mathbf{X}''(t) = \left(Q_1'(t), \ldots, Q_n'(t)\right).$$

The speed of the economy can be measured by the following two norms with unit *€/year* of vector **X'**(*t*):

$$(1)\ speed(t) = \left\|\mathbf{X}'(t)\right\| = \left(Q_1^2(t) + \ldots + Q_n^2(t)\right)^{1/2}$$

$$(2)\ GDP(t) = \left\langle \mathbf{X}'(t) \right\rangle = \left|Q_1(t)\right| + \ldots + \left|Q_n(t)\right|.$$

The corresponding annual growth measures are:

$$\Delta speed(t) = \left(\sum_{i=1}^{n} Q_i^2(t)\right)^{1/2} - \left(\sum_{i=1}^{n} Q_i^2(t-y)\right)^{1/2},$$

where y = *year*, and

$$\Delta GDP(t) = \sum_{i=1}^{n} \Delta Q_i(t) \quad \text{because } Q_i(t) > 0.$$

Conclusions 1:

- From the vector-function describing the position of an economy, we calculate scalar quantities that measure the aggregate speed of the ”point economy” in the *n*-space of accumulated productions in nominal and real terms.
- The velocity of the ”point economy” is a vector quantity, and all norms of the velocity vector are scalar measures for the speed of the ”point economy”.
- *speed* is the Euclidean norm of the velocity vector of the ”point economy”, and Gross Domestic Product (*GDP*) is its absolute value norm. Thus a change in *speed* and a change in *GDP* both measure the growth of the economy.
- We can identify the speed and change of speed of an economy with its flow of production and its growth.

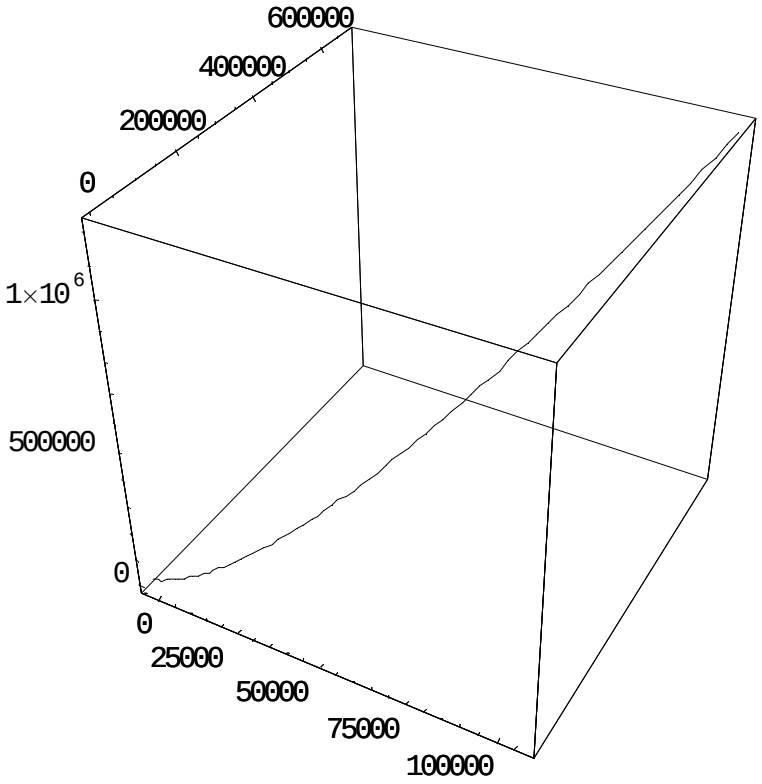


Figure 2a. Three-dimensional fitted accumulated production values in Finland at 1975-2002

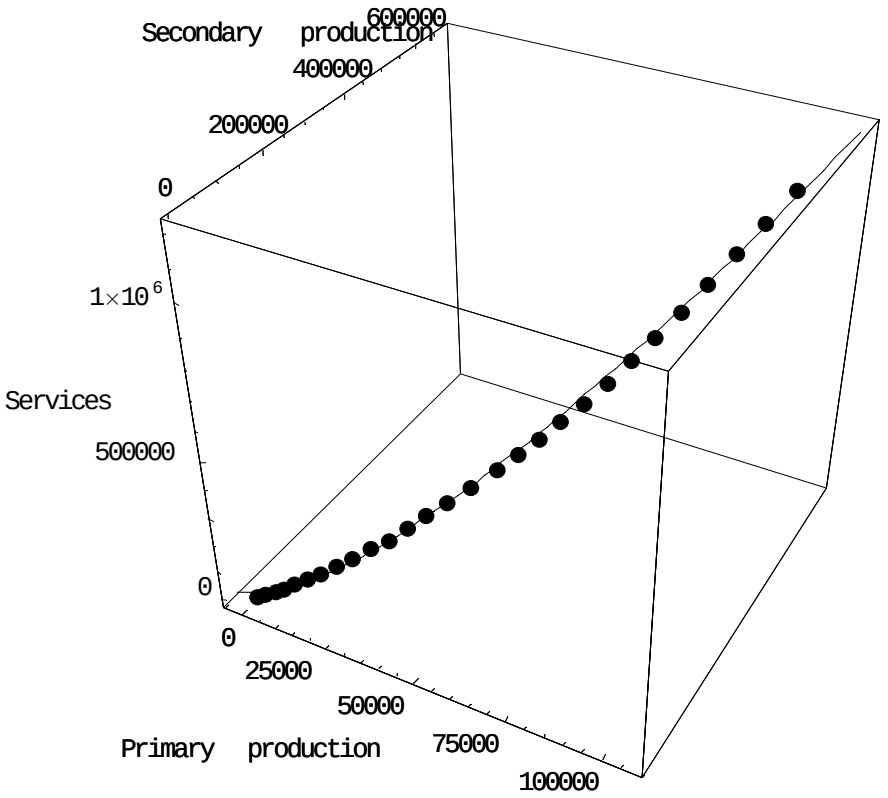


Figure 2b. Three-dimensional actual and fitted accumulated production values in Finland at 1975-2002

In Figure 2a, the following estimated coordinate functions were applied:

$$F_1 = -3391 + 2577\,t + 37\,t^2,$$

$$F_2 = -1229 + 5623\,t + 564\,t^2,$$

$$F_3 = 9934 + 1641\,t + 1278\,t^2,$$

where subindex 1 refers to primary, 2 to secondary production and 3 to services, and t is time.

The following coordinate functions were estimated for accumulated production volumes (values at 2000 prices):

$$F_1 = -2001 + 4545\,t - 15\,t^2,$$

$$F_2 = 894 + 17389\,t + 323\,t^2,$$

$$F_3 = -12022 + 34676\,t + 641\,t^2,$$

where subindex 1 refers to primary, 2 to secondary production and 3 to services, and t is time.

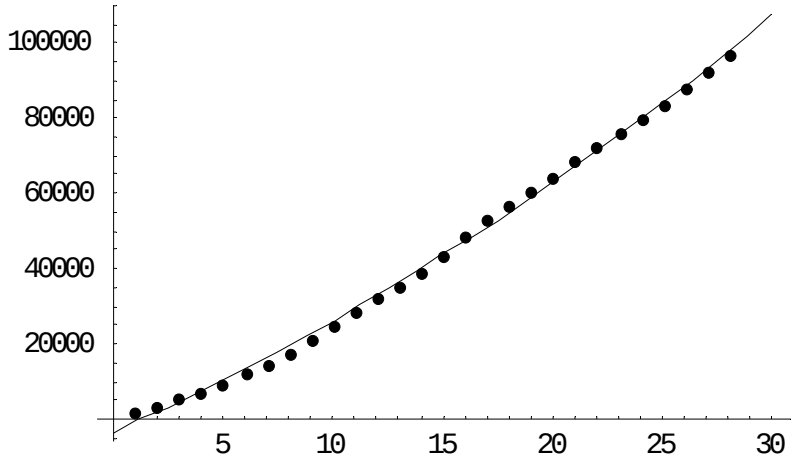
Var.	Estimate	T-stat.
1	-2001.4	-4.2
t	4544.8	60.6
t^2	-15.3	-6.1
Eq. 1	R^2=0.999	F=25704

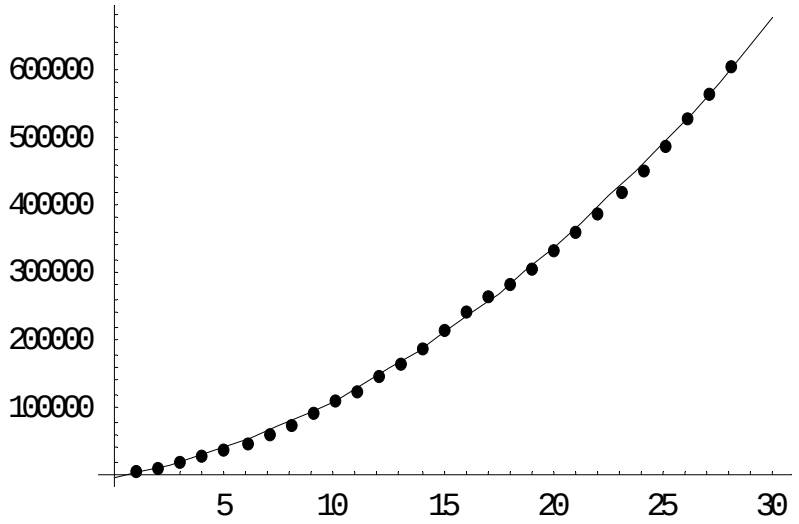
Var.	Estimate	T-stat.
1	894.3	0.3
t	17389.0	38.4
t^2	322.7	21.3
Eq. 2	R^2=0.999	F=30140

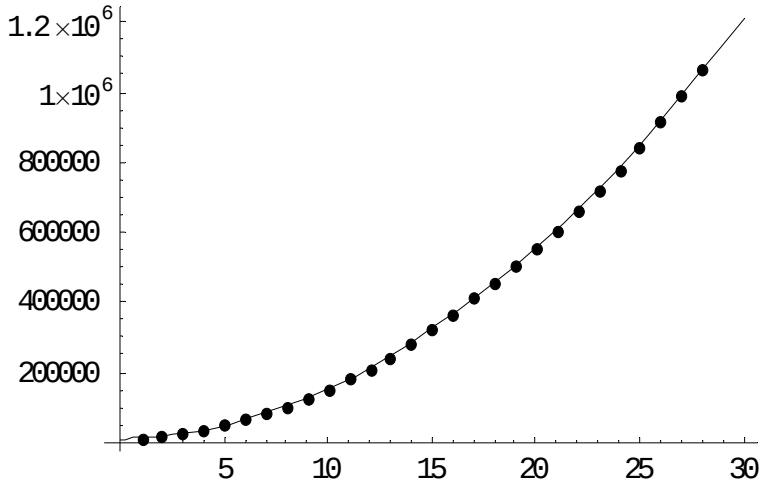
Var.	Estimate	T-stat.
1	-12021.8	-2.3
t	34675.5	41.4
t^2	640.8	22.9
Eq. 3	R^2=0.999	F=34984

The statistical properties of the estimated models for accumulated sectoral production volumes

Models for accumulated sectoral production volumes:







Figures 3a,b,c. Actual and fitted accumulated production volumes in primary (up left) and secondary production (up right), and in services in Finland at 1975-2002

The estimated models for volumes are of the form:

$$\int_0^t Q_i(s)ds = a_i + b_i t + c_i t^2, i = 1,2,3,$$

i.e. $Q_i(t) = b_i + 2c_i t$ *and* $Q_i'(t) = 2c_i$.

Because $Q_i'(t)$ with unit *€/year*^{2} is the acceleration of the value of production, we can interpret the constants c_i in terms of the following Newtonian formulation:

$$F_i = m_i a_i, a_i = Q_i'(t) \Leftrightarrow F_i / m_i = Q_i'(t).$$

Assuming F_i to be constant, the estimated constant c_i is the ratio of force F_i and inertial mass m_i divided by 2, i.e.

$$c_i = F_i /(2m_i).$$

Conclusions 2:

• Because the inertial factors m_i are nonnegative, the force acting upon production has been negative in primary production, and positive in secondary production and in services.

• The force/mass –ratio has been the greatest in services.

•The estimated non-zero accelerations for sectoral productions are in conflict with the neoclassical theory that assumes firms to produce always the optimal amounts.

• The model fits well in the data and can be used in forecasting future development.

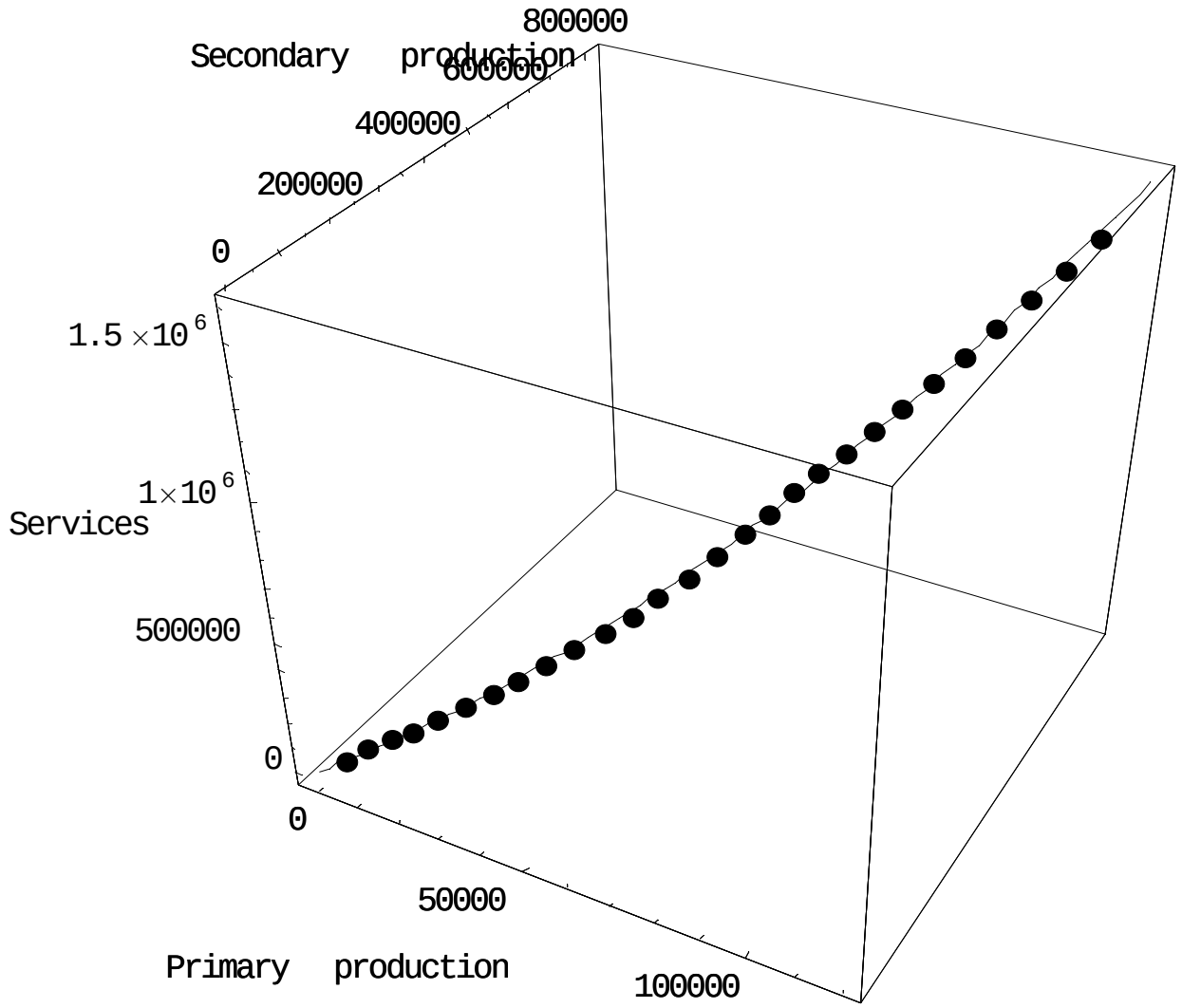


Figure 4. Actual and fitted three-dimensional accumulated production volumes in Finland at 1975-2002

The *GDP* and *speed* -measures in value terms at level and difference forms measured from 9 sectors

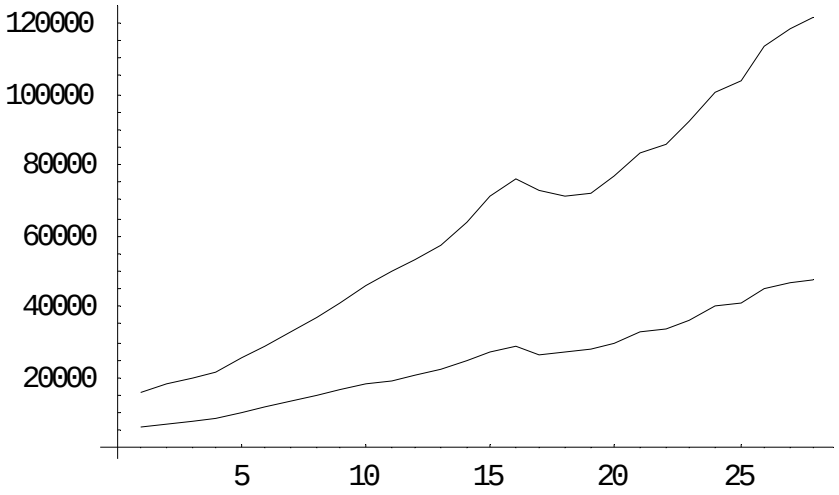


Figure 5. Levels of *GDP* and *speed* in value terms

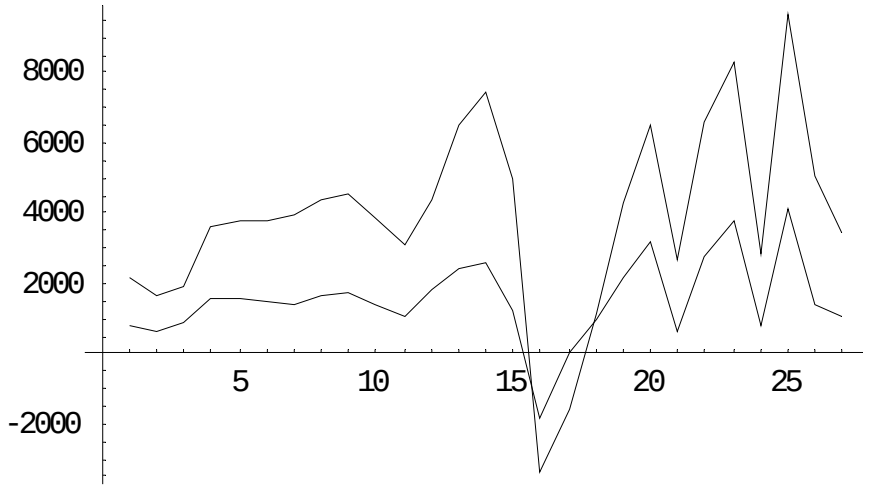


Figure 6. Differenced *GDP* and *speed* in value terms

The *GDP* and *speed* -measures in volume terms at level and difference forms measured from 9 sectors

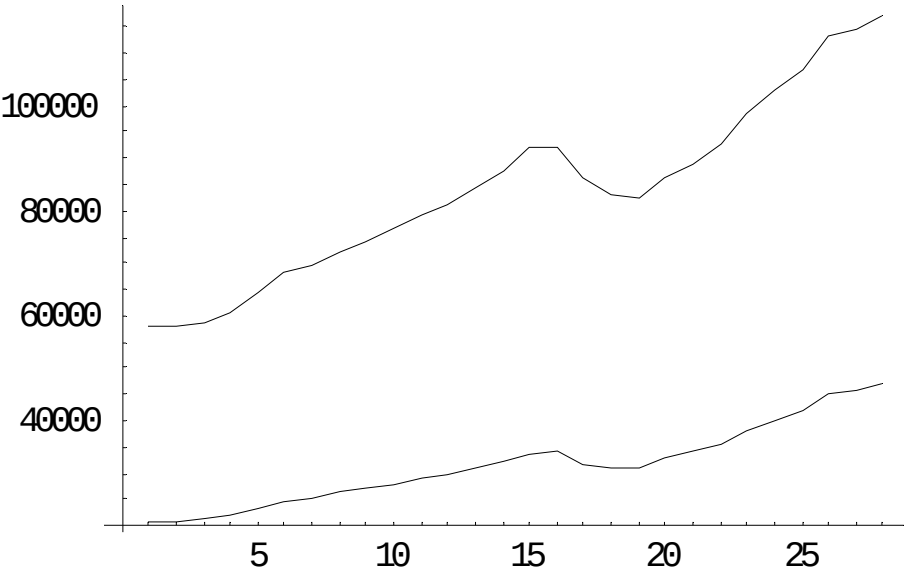


Figure 7. Levels of *GDP* and *speed* in volume terms

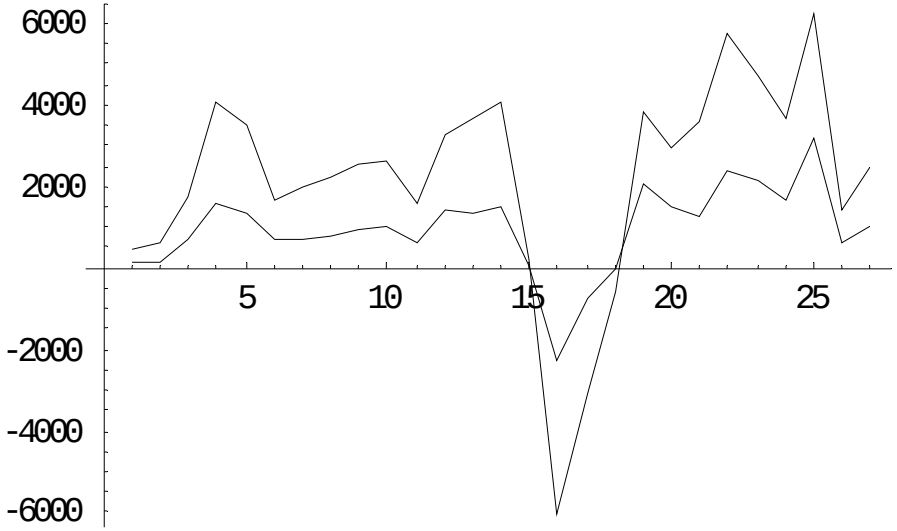


Figure 8. Differenced *GDP* and *speed* in volume terms

Correlations between the production measures

	GDP_n	$speed_n$	GDP_r	$speed_r$
GDP_n	1			
$speed_n$	0.999	1		
GDP_r	0.985	0.986	1	
$speed_r$	0.987	0.999	0.997	1

Subscript *n* refers to nominal and *r* to real data.

Correlations between the growth measures

	ΔGDP_n	$\Delta speed_n$	$\Delta\ GDP_r$	$\Delta\ speed_r$
$\Delta\ GDP_n$	1			
$\Delta\ speed_n$	0.950	1		
$\Delta\ GDP_r$	0.852	0.833	1	
$\Delta\ speed_r$	0.852	0.871	0.978	1

Subscript *n* refers to nominal and *r* to real data.

Part 2:Analysis of structural change

The structure of production is described as:

$$\mathbf{V}(t)=\left(V_1(t),\ldots,V_n(t)\right)=\left(\frac{Q_1(t)}{\sum_{i=1}^{n}Q_i(t)},\ldots,\frac{Q_n(t)}{\sum_{i=1}^{n}Q_i(t)}\right),$$

and our measure *v* for structural change measures the change in this structure by the norm of the velocity vector **V'***(t)* as:

$$v(t)=\left\|\mathbf{V}'(t)\right\|=\left(\left(V_1'(t)\right)^2+\ldots+\left(V_n'(t)\right)^2\right)^{1/2}.$$

We measure the nonlinearity in the expansion of production of the economy by the curvature *k* of the accumulated production path:

$$k(t) = \left\| \frac{\mathbf{T}'(t)}{\mathbf{X}'(t)} \right\|, \quad \text{where} \quad \mathbf{X}'(t) = \left(Q_1(t), \ldots, Q_n(t)\right)$$

$$\text{and} \quad \mathbf{T}(t) = \frac{\mathbf{X}'(t)}{\left\|\mathbf{X}'(t)\right\|} = \left(\frac{X_1'(t)}{\left\|\mathbf{X}'(t)\right\|}, \ldots, \frac{X_n'(t)}{\left\|\mathbf{X}'(t)\right\|} \right).$$

Measure *k* measures the curvature of the space curve **X***(t)* at time moment *t*.

The measures for structural change and nonlinearity of the production path in value terms from 9 sectors

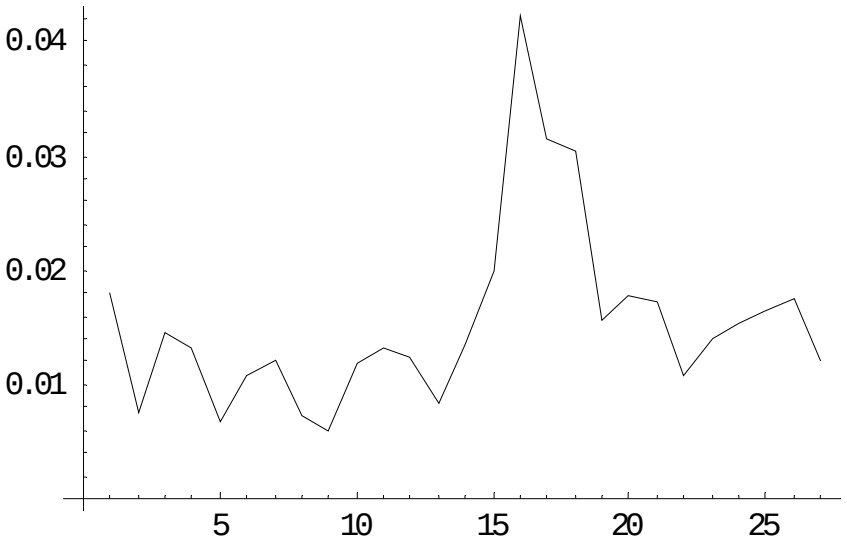


Figure 9. Measure *v* in value terms at 1975-2002

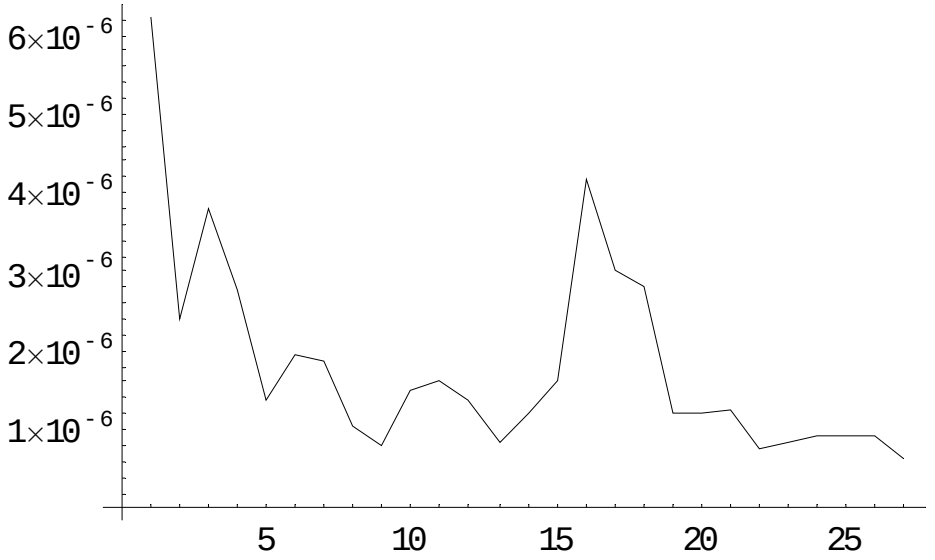


Figure 10. Measure *k* in value terms at 1975-2002

The measures for structural change and nonlinearity of the production path in volume terms from 9 sectors

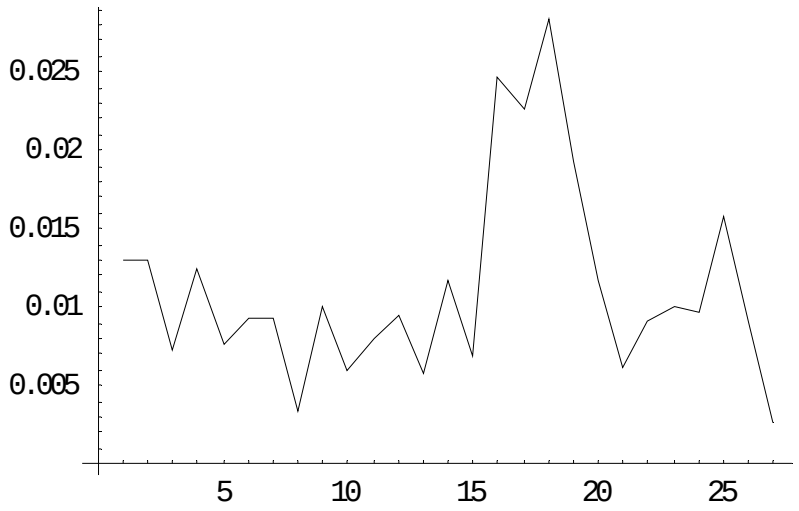


Figure 11. Measure *v* in volume terms at 1975-2002

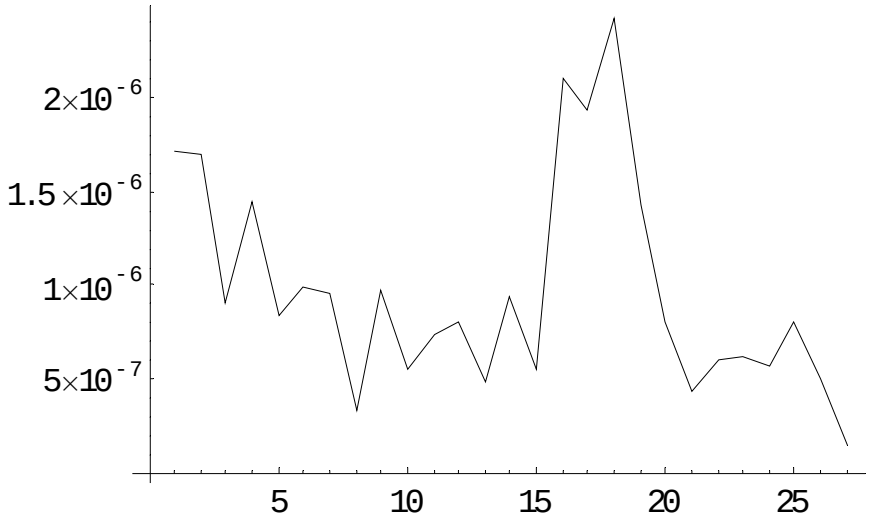


Figure 12. Measure *k* in volume terms at 1975-2002

The measures for structural change and nonlinearity of the production path scaled at equal level

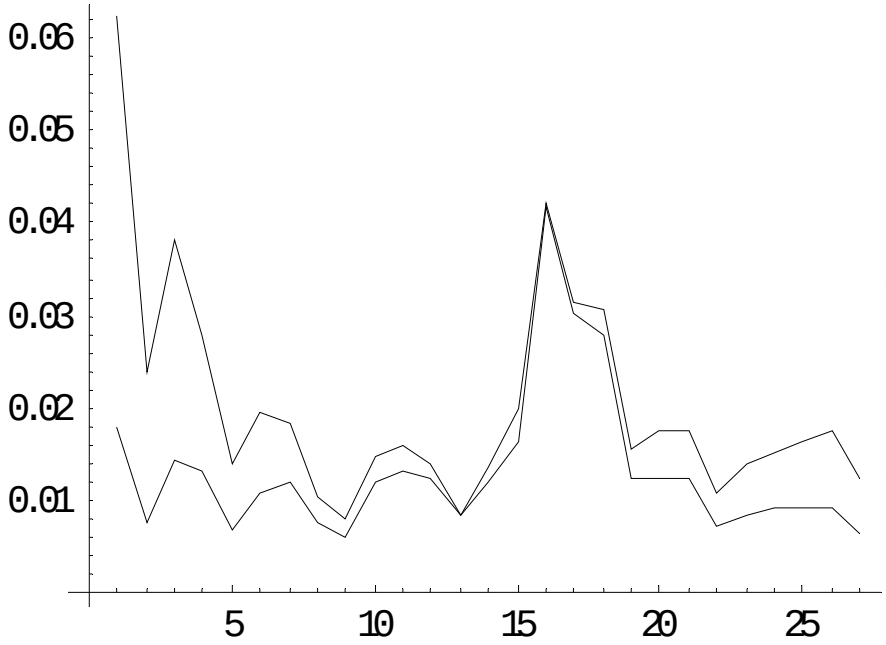


Figure 13. Measures *v* and *k* in value terms scaled at equal level

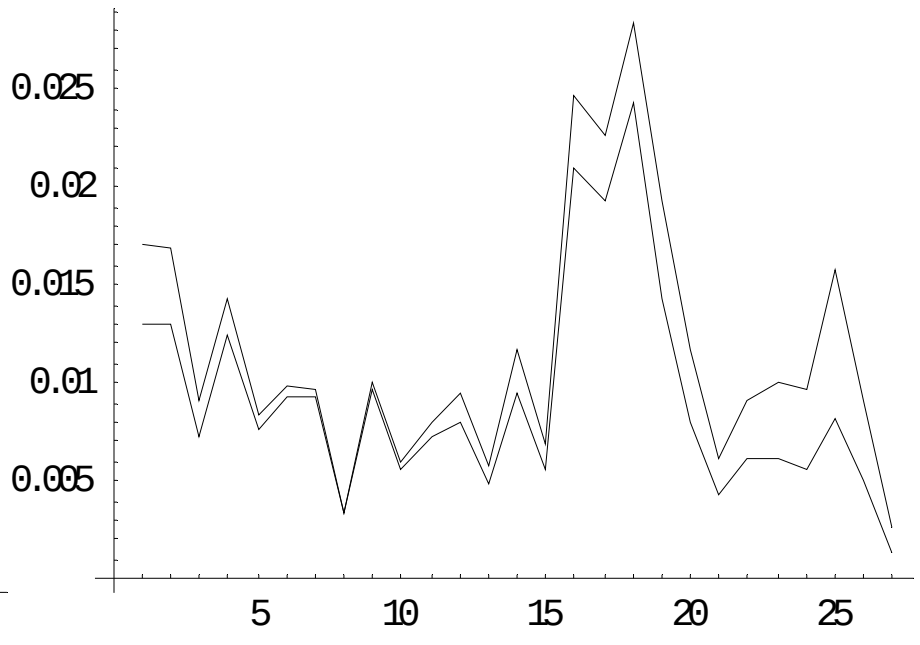


Figure 14. Measures *v* and *k* in volume terms scaled at equal level

Correlations between the structural change measures

	v_n	k_n	v_r	k_r
v_n	1			
k_n	0.494	1		
v_r	0.767	0.446	1	
k_r	0.628	0.699	0.898	1

Subscript *n* refers to nominal and *r* to real data.

Conclusions 3:

- If the production structure of an economy is described by the shares of sectors of total production, the structure can be described by a vector of the shares of sectoral productions.
- Changes in the structure can be measured by any norm of the velocity vector of the production structure vector.
- The amount of nonlinearity of the production path of an economy can be measured by the curvature of the path.
- These two measures are highly correlated.

Annual value data from 9 sectors in Finland at 1975-2002

1644	4810	1657	2169	1296	477	1688	1481	893
1816	5413	1629	2458	1616	576	1911	1762	1120
1926	5882	1716	2592	1807	650	2115	1950	1285
1967	6685	1749	2847	2013	699	2340	2095	1444
2223	8066	1921	3332	2374	790	2641	2371	1696
2617	9257	2195	3929	2633	1002	3010	2697	1883
2707	10309	2457	4465	2989	1136	3560	3148	2215
3009	11192	2788	5048	3243	1268	4154	3601	2634
3335	12282	3244	5586	3653	1379	4684	4128	3014
3636	13547	3491	6174	4103	1573	5283	4511	3527
3761	14361	3549	6852	4493	1819	5745	5035	4115
3615	14878	3831	7167	4846	2096	6457	5468	4498
3314	16344	4143	8003	5215	2388	6952	5989	4841
3823	17926	4944	8922	5771	2607	7885	6669	5171
4363	19485	6279	9878	6369	2937	8854	7344	5648
4990	19627	6547	10186	7030	3656	9625	8065	6428
4118	16781	5636	9276	7059	3235	10315	8856	7464
3693	17272	4206	8435	7110	2500	10706	8855	8374
3921	18888	3474	8374	7278	3166	11390	8532	7285
4088	21012	3579	9081	7680	3020	12064	8674	7366
3876	24072	3852	9806	8162	3358	12951	9051	7900
3626	23662	4279	10312	8630	3300	13786	9467	8670
3997	25608	4682	11513	9340	3392	15287	9764	8728
3669	28936	5547	12426	10294	3631	16523	10199	9339
3850	28625	6116	12647	10924	3275	17820	10593	9591
4399	32413	6445	13048	12090	4475	19250	11067	9868
4225	32140	6829	13957	12811	4646	20873	11678	10947
4401	31949	6700	14375	13308	4628	22062	12208	11913

Annual volume data from 9 sectors in Finland at 1975-2002

3595	11967	6744	8135	4826	1900	7386	7378	5818
3682	12007	6217	8126	4736	2015	7564	7623	6257
3840	11957	6308	7581	4745	2091	7865	7864	6583
3913	12497	6232	7716	4881	2115	8241	8003	7008
4423	13883	6220	8292	5307	2145	8731	8305	7363
4680	15006	6655	8591	5552	2377	9241	8556	7554
4467	15629	6522	8548	5681	2644	9593	8912	7879
4367	15755	6817	8980	5704	2534	10170	9111	8408
4502	16171	7100	9189	5835	2762	10606	9365	8577
4722	16996	6827	9623	5992	3156	11119	9354	8846
4675	17761	6876	10059	6206	3512	11447	9503	9246
4338	18040	6912	10465	6316	3843	11979	9606	9362
4113	19121	6965	11239	6747	4101	12568	9858	9422
4374	19993	7523	11874	7137	4116	13117	10081	9553
4127	20800	8554	12877	7620	4410	13574	10284	9602
3830	20780	8438	12641	8055	4459	13768	10358	9755
3338	18550	7499	11101	7754	3430	13672	10468	10193
3470	18590	6673	9466	7759	2639	13640	10271	10427
3637	19510	5883	9050	7990	3488	14015	9992	8765
3980	21761	5497	9642	8309	3088	14665	10049	9179
3749	23167	5133	10292	8746	3148	15269	10196	9387
3720	23919	5706	10639	9254	3465	15864	10376	9731
4184	25995	6239	11213	9954	3662	16605	10669	9928
3878	27740	6736	12099	10773	3770	17376	10768	10056
3976	29200	6616	12587	11276	4263	18258	10938	9745
4399	32413	6445	13048	12090	4475	19250	11067	9868
4189	32505	6258	13482	12349	3996	19899	11215	10548
4341	33202	6417	13728	12697	3949	20556	11390	10664