Innovation, Imitation and Investment Waves through Local and and Global Dynamics

Massimo Ricottilli
University of Bologna
massimo.ricottilli@unibo.it

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1 Introduction

1.1 Technical progress

- Technical progress as an endogenous engine of growth: a prime mover that explains a major source of effective demand.

  Technical progress, however, comes in lapses: innovation-waves occur.

  Technical progress does not fall as *manna from heaven* but is the outcome of a searching process

  The paper attempts to relate technical progress and its likely pattern to investment

  It exclusively concentrates on technology-driven investment

2 The traverse, made simple

Investment that modifies the economy's structure creates discontinuities

It generates waves of effective demand

This analysis develops through a time partition

Four periods:

-the state of normality

-innovation and investment: the construction period

-investment wanes as new capital goods appear

-imitation

3 The state of normality

- It is the benchmark against which to assess the progress of traverse.
- It is a stationary state, to simplify population growth is zero
- The last innovation wave is *long* past and all there was to imitate has been imitated
- There is no real investment on the assumption that capital goods are everlasting
- Only consumption goods are produced
- Firms earn an historically given profit rate: $\overline{r}$
- However, there is investment in the search-for-innovation process

3.1 The economy, made simple

- n firms in J industries, n_j firms in each industry
- fimrs and industries are vertically integrated: they produce their own capital goods.
- a simple Leontieff-type production of consumption goods function: $y_j = \min(\frac{1}{a_j} L_j, K_j)$
- labour coefficient, a_j. Capital goods are produced by labour alone with b_j units of labour.
- Firms' output and capital stock normalised at 1.
- Total output of consumption goods: $C = \sum_{j=1}^{J} n_j p_j$
- Employment in the consumption goods industries: $L_c = \sum_{j=1}^{J} n_j a_j$
- Investment in the searching process: $\Pi_j(\overline{\tau}) = w_I L_{Ij} \frac{e^{\overline{r}\overline{\tau}} - 1}{\overline{r}}$

3.2 Prices

- Price of consumption good j :
- $$p_j = w(a_j + b_j\overline{r}), \qquad \forall j$$
- Prices of capital goods:

$$q_j = wb_j$$

3.3 Effective demand

- Investment:

$$I_{Ij} \equiv \frac{d\pi_j}{dt} = w_I L_{Ij} e^{\bar{r}t}$$

- Consumption: only profit earners save

$$C = w\left(\omega \sum_{j=1}^{J} L_{Ij} + \sum_{j=1}^{J} n_j a_j\right) + (1 - s_\pi) w\bar{r} \sum_{j=1}^{J} n_j b_j = \sum_{j=1}^{J} n_j p_j$$

Consumption demand in each industry

$$n_j p_j = \beta_j C$$

- Aggregate demand and supply, savings and invetsment equilibrium:

$$s_\pi \overline{r} \sum_{j=1}^{J} n_j b_j = \omega \sum_{j=1}^{J} L_{Ij}$$

- The propensity to save :

$$s_\pi = \frac{\omega \sum_j L_{Ij}}{\overline{r} \sum_j n_j b_j}$$

4 Breaking the state of normality

Assumption 1: *Productivity is assumed to rise as follows:*

$$a_j^* = a_j e^{-\lambda}$$

$$b_j^* = b_j e^{-\lambda}$$

Investment takes place iff $r_T > \overline{r}$, given that H_j is the expected time span to gain extra profits

$$b_j^* w + \Pi_j(T_j) = \phi_j \left[(p_j - w a_j^*) E \left(\int_{T_j}^{T_j + H_j} e^{-r_T \tau} d\tau \right) + (p_j^* - w a_j^*) E \left(\int_{T_j + H_j}^{+\infty} e^{-\overline{r}\tau} d\tau \right) \right]$$

4.1 The technology cross-over

New capital goods are manufactured: the construction phase,conventionally, lasts one period.

- The macroeconomic equilibrium now required:

$$\sum_{j=1}^{J} n_j p_j^* = w\left(\omega \sum_{j=1}^{J} L_{Ij} + \sum_{j=1}^{J} n_j a_j + \sum_{k=1}^{h^*} b_k^*\right) + (1 - s_\pi) r^* w \sum_{J=1}^{J} n_j b_j$$

- The equilibrium profit rate

$$r^* = \frac{\omega \sum_j L_{Ij} + \sum_{k=1}^{h^*} b_k^*}{s_\pi \sum_j n_j b_j}$$

4.2 Adjustment to aggregate supply and demand imbalance

- Disequilibrium is essentially macroeconomic: it is resolved by
- $$\dot{r} = \zeta \left[\left(\omega \sum_{j=1}^{J} L_{Ij} + \sum_{k=1}^{h^*} b_j^* \right) - s_\pi r \sum_{j=1}^{J} n_j b_j \right]$$
- Basically: $$\dot{r} = \zeta \left[I - S\left(r \right) \right]$$
- Prices rise, (nominal wages are assumed fixed)
- Real wages fall
- Employment rises

1.

4.3 The construction phase ends: second traverse period

- Unemployment rises as workers engaged upstream are laid off
- Demand for capital goods now wanes and investment is back to R.&D.
- Innovators earn a monopolist profit rate r_T^{**} whilst laggards earn:

$$r_j^{**} = \left[r_T^{**} - \left(e^{\lambda} - 1 \right) \frac{a_j}{b_j} \right] e^{-\lambda}$$

- r_T^{**} is an equilibrium magnitude determined by

$$\omega \sum_{j=1}^{J} L_{Ij} = s_{\pi} \left[r_T^{**} \sum_{j=1}^{J} n_j b_j^* - \left(1 - e^{-\lambda} \right) \left(\sum_{j=1}^{J} a_j n_j - \sum_{k=1}^{h^*} a_k \right) \right]$$

4.4 Macroeconomic adjustment

- $r_T^{**} \gtreqless r^*$ iff

$$\left(e^{\lambda}-1\right)\left[\omega\sum_{j=1}^{J}L_{Ij}+s_{\pi}(\sum_{j=1}^{J}n_j a_j-\sum_{k=1}^{h^*}a_k)\right]\gtreqless\sum_{k=1}^{h^*}b_k^*$$

- prices may actually fall in account of a lack of efefctive demand and real wages rise

4.5 Searching to innovate

1.
   - Leading firms carry out a research process
   - They employ specialised manpower (brainpower) L_{Ij}. $L_{Ij} = L_{I_1 j} + L_{I_2 j}$
   - $h_j = h(L_{I_1 j})$; $h'_j > 0$; $h''_j < 0$. Average: $h = \frac{1}{J}\sum_{j=1}^{J} h_j(L_{I_1 j})$
   - $\epsilon_j = \epsilon(L_{I_2 j}, J)$; $\epsilon_L > 0$ and $\epsilon_J > 0$.Average: $\hat{\epsilon} = \frac{1}{J}\sum_{j=1}^{J} \epsilon(L_{I_2 j}, J)$.
   - $\epsilon_J \gtreqless 1$
   - The importance of J

5 Master equations

- $$\begin{aligned}
\dot{\rho}_S &= -\rho_S + (h + S\hat{\epsilon}\rho_S)\,\rho_{S-1} \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
\dot{\rho}_k &= -(h + S\hat{\epsilon}\rho_S)\,\rho_k + (h + S\hat{\epsilon}\rho_S)\,\rho_{k-1} \\
\dot{\rho}_{k-1} &= -(h + S\hat{\epsilon}\rho_S)\,\rho_{k-1} + (h + S\hat{\epsilon}\rho_S)\,\rho_{k-2} \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
\dot{\rho}_0 &= -(h + h + S\hat{\epsilon}\rho_S)\,\rho_0 + \rho_S
\end{aligned}$$

5.1 Avalanche and waiting time

- In the limit $h \to 0$;
- Symmetry: $L_{I_1 j} = L_{I_1}$, $L_{I_2 j} = L_{I_2}$, $L_{I_1} + L_{I_2} = L_I$, $\forall i, j$
- The avalanche size:

$$E(V_T) = \frac{1}{1 - \epsilon(L_I - L_{I_1}, J)S}$$

- Waiting time between innovation waves

$$\varphi \approx \frac{S}{h}$$

5.2 Profit expectations

1.
   - The probability of an informative shock:

$$\pi_j = h(L_{I_1 j}) + \epsilon(L_{I_2 j}, J) S \frac{h}{1-\hat{\epsilon}}$$

   - The probability that a firm in a critical state innovate:

$$\phi(L_I) = \rho_{S-1}\pi(L_I)$$

   - Solve for L_I, given that $\Pr(H_j \leq \tau) = (1 - e^{-\mu\tau})$

     $b_j^* w + \Pi_j(T_j) = \phi_j \left[(p_j - w a_j^*) E \left(\int_{T_j}^{T_j+H_j} e^{-r_T \tau} d\tau \right) + (p_j^* - w a_j^*) E \left(\int_{T_j+H_j}^{+\infty} e^{-\bar{r}\tau} d\tau \right) \right]$

     such that $r_T = \max$
   - Given a solution L_I, firms maximise π_j ; f.o.c. $h'(L_{I_1})\frac{1-\hat{\epsilon}}{h} = \epsilon'(L_I - L_{I_1}, J)S$ from which L_{I_1} and L_{I_2} .

5.3 Imitation

- Laggards imitate
- Imitation is a Poisson event with arrival rate μ.
- The share of firms that from H onwards are able to get to the frontier is s_k
- Hence in each period
$$\dot{s}_k = \mu n_k s_k (1 - s_k)$$
- Solving for
$$s_k = \frac{1}{1 + (n_k - 1)e^{-\mu n_k t}}$$

5.4 Imitation-driven Investment

- Investment

$$I_k = b_k^* \dot{s}_k n_k$$

- Macroeconomic equilibrium

$$\omega \sum_{j=1}^{J} L_{Ij} + \sum_{k=1}^{h^*} b_k^* \dot{s}_k n_k = s_\pi r_T(t) \sum_{j=1}^{J} n_j b_j^* - s_\pi (1 - e^{-\lambda}) \left(\sum_{k=1}^{h^*} (1 - s_k) n_k a_k + \sum_{j \neq k}^{J} n_j a_j \right)$$

- The equilibrium profit rate

$$r_T(t) = \frac{\omega \sum_{j=1}^{J} L_{Ij} + \sum_{k=1}^{h^*} b_k^* \dot{s}_k n_k + s_\pi (1 - e^{-\lambda}) \left(\sum_{k=1}^{h^*} (1 - s_k) n_k a_k + \sum_{j \neq k}^{J} n_j a_j \right)}{s_\pi \sum_{j=1}^{J} n_j b_j^*}$$

5.5 Conclusions

- Innovation and diffusion of innovative information generates waves of investment.
- This is due to both autonomous, in-house, R&D and, crucually, to interaction among firms; firms are rationality bounded.
- The wave-like pattern of technology-driven investment is an underlying cause of instability. It generates upwings and downswings both in employment and prices.
- The cross-over from old to new techniques engenders a Hicksian traverse problem
- The imbalance is basically macroeconomic: adjustment procedes from the macro-level to the micro-level.
- Expectations are frustrated because firms cannot forecast the traverse turmoil and the related oscillations in profit rates.
- Imitation is another source of investment carried out by laggard firms: it gradually restores normality before a new wave sets in.