

# A Heterogeneous Agents Model with a Market Maker: Theory and Evidence

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By

Carl Chiarella, University of Technology, Sydney  
Xue-Zhong He, University of Technology, Sydney  
Remco C.J. Zwinkels, Erasmus Universiteit Rotterdam

# Outline

- Heterogeneous Agents Models
- Empirical Evidence so far
- Our Contribution
- Model
- Methodology
- Estimation Results
- Simulation Results
- Conclusion

# Introduction: heterogeneous agents models (HAMs)

- Asset pricing models with bounded rationality:
  - Price is weighted average of boundedly rational price forecasts
  - typically: fundamentalists and chartists
  - Switching between groups conditional on past performance
  - Hommes (2006); Day and Huang (1990), Lux (1998) Brock and Hommes (1997,1998), Chiarella and He (2002)

# Introduction: HAMs and Empirics

- Generally: analytical or simulation studies
  - Deterministic / Stochastic properties
  - Stylized facts
- Estimation evidence limited due to
  - non-linearity of models
  - functional form of agents
  - Frankel and Froot (1986)
  - Vigfusson (1997), Ahrens and Reitz (2005)
  - Baak (1999), Chavas (2000)
  - Westerhoff and Reitz (2003), Reitz and Westerhoff (2006), Manzan and Westerhoff (2007)
  - Winker and Gilli (2001, 2004)
- Boswijk, Hommes, Manzan (2007)

# This paper

- Direct estimation of Heterogeneous Agents Model
  - Theory:
    - ~ extension of Chiarella and He (2003)
    - ~ third group: noise traders
    - ~ market maker
    - ~ more flexible / realistic chartist behavior
  - Evidence:
    - ~ direct estimation of HAM with switching
    - ~ close to model
    - ~ high(er) frequency data

# Model (1)

- Framework from Chiarella and He (2003)
- basis is dynamic Gordon growth model

$$E_t(y_{t+1}) = (1 + g_t)y_t$$

$$\bar{p}_t = \frac{(1 + g_t)}{rr_t - g_t} y_t, \quad g_t = E(g | X_t), rr_t = E(rr | X_t),$$

# Model (2)

- Define

$$x_t = p_t - \bar{p}_t$$

- Return becomes

$$R_t = x_t - Rx_{t-1} + \delta_t$$

- In which

$$\delta_t = \bar{p}_{t+1} + y_{t+1} - E[\bar{p}_{t+1} + y_{t+1}], \quad E_t[\delta_{t+1}] = 0$$

# Model (3)

- Expectations by fundamentalists are then given by

$$E_{f,t}(x_{t+1}) = x_t + \alpha x_t = (1 + \alpha)x_t$$

$$E_{f,t}(R_{t+1}) = (1 - R + \alpha)x_t$$

- and chartists

$$E_{c,t}(x_{t+1}) = x_t + d(\tau_t - x_t)$$

$$\tau_t = \omega\tau_{t-1} + (1 - \omega)x_t$$

$$E_{c,t}(R_{t+1}) = (1 - R)x_t + d(\tau_t - x_t)$$

# Model (4)

- Demand for fundamentalists, chartists, and noise traders is then given by

$$z_{f,t} = \frac{(1-R+\alpha)x_t}{a\sigma^2}$$

$$z_{f,t} = \frac{(1-R)x_t + d(\tau_t - x_t)}{a\sigma^2}, \quad \tau_t = \omega\tau_{t-1} + (1-\omega)x_t$$

$$z_{n,t} = \sqrt{S_t}\varepsilon_t$$

$$S_t = \phi_1 + \phi_2 S_{t-1} + \phi_3 \left( \sqrt{S_{t-1}}\varepsilon_{t-1} \right)^2, \quad \varepsilon_t \sim N(0,1)$$

# Model (5)

- Demand is submitted to market maker, who adjusts prices accordingly

$$x_{t+1} = x_t + \mu \left( n_{f,t} z_{f,t} + n_{c,t} z_{c,t} + n_n z_{n,t} \right)$$

- define

$$m_t = \frac{n_{f,t} - n_{c,t}}{n_{f,t} + n_{c,t}} = \frac{n_{f,t} - n_{c,t}}{1 - n_n}$$

- then

$$x_{t+1} = x_t + \frac{\mu}{2} \left( (1 - n_n) \left[ (1 + m_t) z_{f,t} + (1 - m_t) z_{c,t} \right] + n_n z_{n,t} \right)$$

# Model (6)

- Profit is given by return times demand

$$\pi_{h,t} = (x_t - Rx_{t-1} + \delta_t) z_{h,t-1}$$

- Relative size of the groups is function of profits

$$m_t = \tanh \left( \frac{\beta}{2} (x_t - Rx_{t-1} + \delta_t) [z_{f,t-1} - z_{c,t-1}] \right)$$

# Model (7)

- Total system is subsequently given by

$$x_{t+1} - x_t = \frac{\mu(1-n_n)}{2a\sigma^2} [2(1-R)x_t + (1+m_t)\alpha x_t + (1-m_t)d(\tau_t - x_t)] + \frac{n_n\mu}{2} \sqrt{S_t} \varepsilon_t$$

$$\tau_{t+1} = \omega\tau_t + (1-\omega)x_{t+1}$$

$$S_{t+1} = \phi_1 + \phi_2 S_t + \phi_3 \left( \sqrt{S_t} \varepsilon_t \right)^2$$

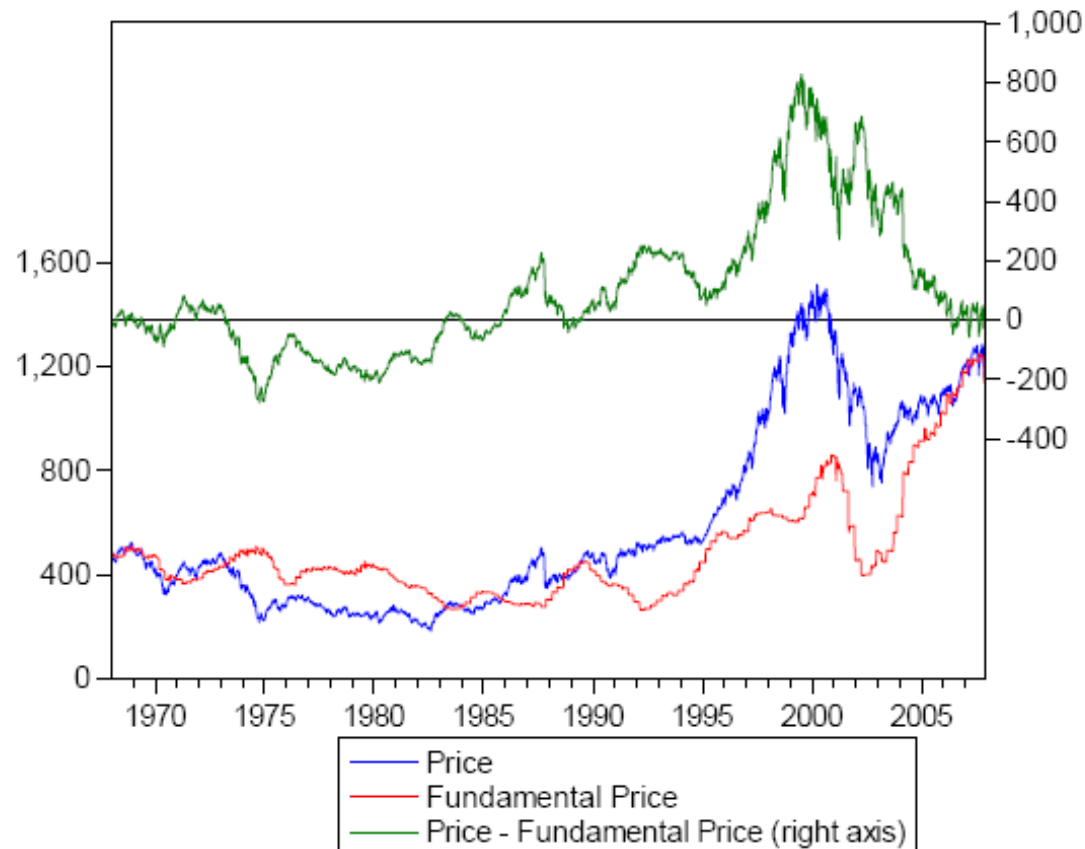
$$m_{t+1} = \tanh \left( \frac{\beta}{2a\sigma^2} (x_{t+1} - Rx_t + \delta_{t+1})(\alpha x_t - d(\tau_t - x_t)) \right)$$

# Data

- Weekly S&P500
- 1968 – 2007; 2082 observation
- Prices and earnings (monthly)
- Deflated using CPI

# Methodology: fundamental price

$$\bar{p}_t = \frac{(1 + g_t)}{E_t(y/p)} y_t$$



# Methodology: econometric model (1)

- Assuming  $R = 1; \frac{1-n_n}{2a\sigma^2} = 1, \frac{n_n\mu}{2} = 1, \delta_t = 0$
- Two-step approach to full estimation
  - Step 1: remove identification issues by assuming fixed weights ( $m = 0$ ) and unit adjustment ( $\mu = 1$ )

$$x_t - x_{t-1} = c + \alpha x_{t-1} + d(\tau_{t-1} - x_{t-1}) + \sqrt{S_t} \varepsilon_t$$

$$\tau_t = \omega \tau_{t-1} + (1 - \omega)x_t$$

$$S_t = \phi_1 + \phi_2 S_{t-1} + \phi_3 \left( \sqrt{S_{t-1}} \varepsilon_{t-1} \right)^2$$

# Methodology: econometric model (2)

- Step 2: Switching

$$x_t - x_{t-1} = c + (1 + m_{t-1})\alpha^* x_{t-1} + (1 - m_{t-1})d^* (\tau_{t-1} - x_{t-1}) + \sqrt{S_t} \varepsilon_t$$

$$\tau_t = \omega \tau_{t-1} + (1 - \omega)x_t$$

$$m_t = \tanh\left((x_t - x_{t-1})(\alpha^{**} x_{t-1} - d^{**} (\tau_{t-1} - x_{t-1}))\right)$$

$$S_t = \phi_1 + \phi_2 S_{t-1} + \phi_3 \left(\sqrt{S_{t-1}} \varepsilon_{t-1}\right)^2$$

# Methodology: econometric model (3)

- Estimated coefficients defined as

$$\alpha^* = \mu\alpha; d^* = \mu d; \alpha^{**} = \beta\alpha; d^{**} = \beta d$$

- Actual coefficients can be retrieved by numerically minimizing

$$Z = \frac{|\alpha^* - \mu\alpha|}{|\alpha^* + \mu\alpha|} + \frac{|d^* - \mu d|}{|d^* + \mu d|} + \frac{|\alpha^{**} - \beta\alpha|}{|\alpha^{**} + \beta\alpha|} + \frac{|d^{**} - \beta d|}{|d^{**} + \beta d|}$$

# Results: Static Model

	Total Sample	1968 - 1988	1989 - 2007
$c$	-0.055 (0.203)	-0.342 (0.288)	1.653*** (0.450)
$\alpha$	-0.004** (0.002)	-0.006* (0.003)	-0.008*** (0.002)
$d$	-0.010 (0.009)	-0.025* (0.015)	0.099 (0.063)
$\omega$	0.968*** (0.368)	0.950*** (0.040)	0.721*** (0.183)
$\phi_1$	0.604*** (0.107)	1.766*** (0.304)	3.116*** (0.281)
$\phi_2$	0.956*** (0.003)	0.925*** (0.007)	0.955*** (0.004)
$\phi_3$	0.044*** (0.004)	0.056*** (0.007)	0.041*** (0.006)
$\log l$	-8220.84	-3890.71	-4318.39
obs	2078	1091	983

# Results: Full Model (1)

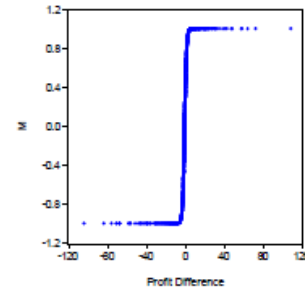
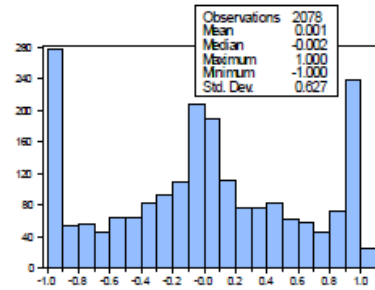
	Total Sample	1968 - 1988	1989 - 2007
	Estimated Coefficients		
$c$	-0.031 (0.221)	-0.248 (0.275)	0.783 (0.583)
$\alpha^*$	-0.004*** (0.001)	-0.004* (0.003)	-0.006*** (0.002)
$d^*$	-0.025** (0.012)	-0.047** (0.019)	-0.008** (0.004)
$\omega$	0.940*** (0.170)	0.921*** (0.158)	0.996*** (0.220)
$\alpha^{**}$	-0.0009 (0.001)	-0.0005 (0.001)	.0001 (0.002)
$d^{**}$	-0.006 (0.006)	0.007 (0.007)	0.003 (0.004)
$\phi_1$	0.309*** (0.091)	1.204*** (0.258)	0.968*** (0.200)
$\phi_2$	0.964*** (0.002)	0.938*** (0.006)	0.972*** (0.003)
$\phi_3$	0.038*** (0.003)	0.051*** (0.006)	0.029*** (0.004)
$\log l$	-8201.59	-3883.93	-4293.84
Obs	2078	1091	983

## Results: Full Model (2)

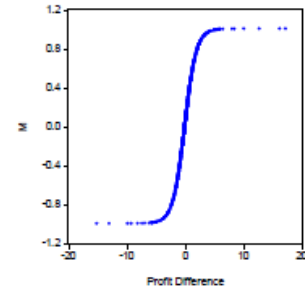
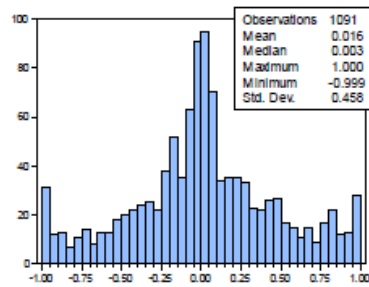
	Total Sample	1968 - 1988	1989 - 2007
	Implied Coefficients		
$\alpha$	-0.004	-0.006	-0.015
$d$	-0.028	-0.064	-0.018
$\mu$	0.886	0.725	0.429
$\beta$	10.138	10.276	14.086

# Results: Weights

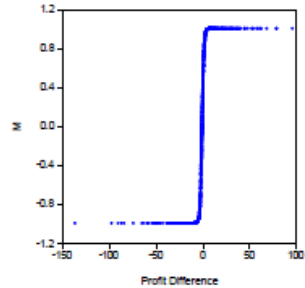
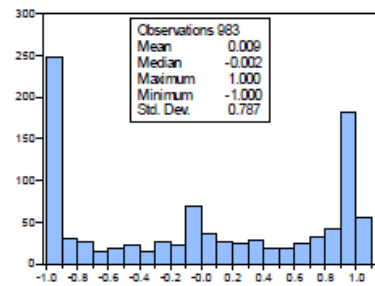
Full Sample



1968-1988



1989-2007

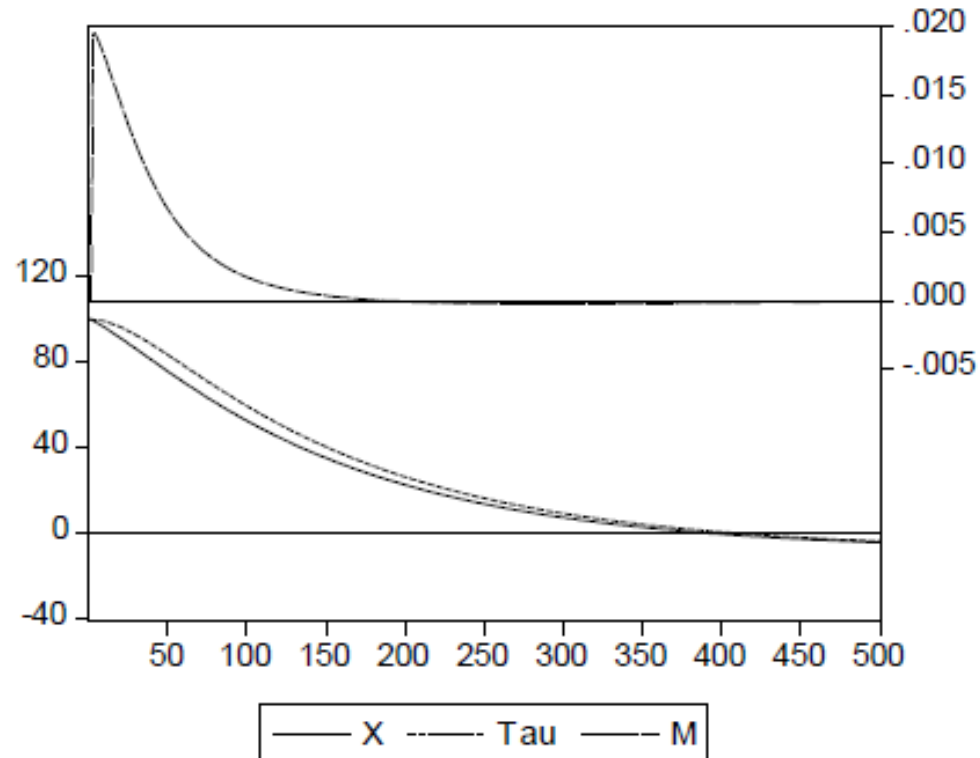


# Simulations

- Deterministic
  - No noise traders
  - Check stability of the model
- Stochastic
  - With noise traders; Monte Carlo simulation
  - Stylized facts
- Coefficients / Starting values

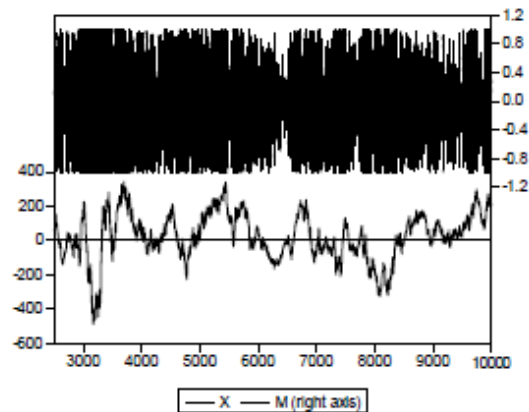
Coefficients		Starting values	
$c$	-0.031	$x_0$	100
$\alpha^*$	-0.004	$\tau_0$	100
$d^*$	-0.025	$m_0$	0
$\omega$	0.940	$S_t$	80
$\alpha^{**}$	-0.0009		
$d^{**}$	-0.006		
$\phi_1$	0.309		
$\phi_2$	0.964		
$\phi_3$	0.038		

# Deterministic Simulations

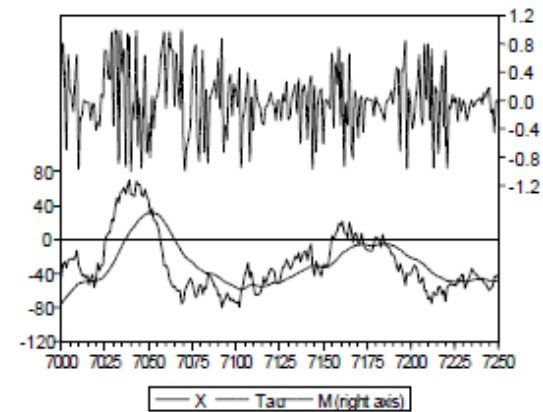


# Stochastic Simulations (1)

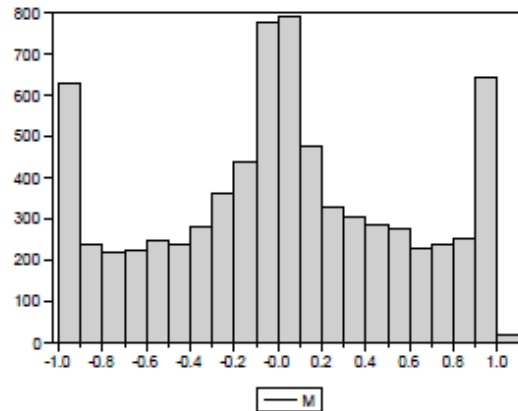
(A)



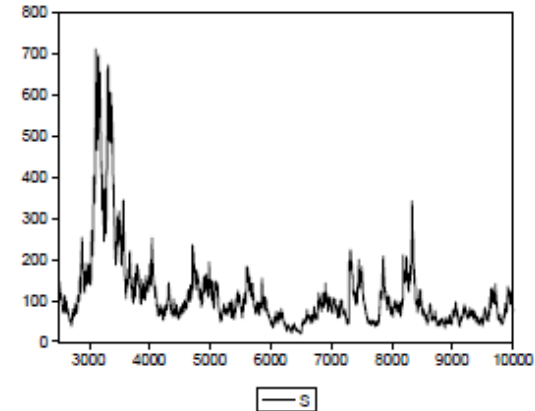
(B)



(C)



(D)



# Stochastic Simulations (2)

- Descriptive statistics of 1,000 runs of 10,000 periods; first 2,500 periods discarded

	Mean $x_t$	Med. $x_t$	St.Dev. $x_t$	Skew $x_t$	Kurt $x_t$	AC $\Delta x_t$	AC $(\Delta x_t)^2$	Unit Root $x_t$
Mean	-.0017	-.0036	12.392	.0003	4.9967	-.0293	0.1774	.0135
Med.	-.0027	.0006	11.834	.0011	4.4377	-.0288	0.1683	.0046
Max	.1647	.3517	30.097	.2530	17.808	.0165	0.4009	.5079
Min	-.1533	-.3585	8.424	-.2771	3.2669	-.0823	0.0647	.0000
St.Dev.	.0295	.1002	2.421	.0659	1.7497	.0154	0.0563	.0308

# Conclusions

- Based on Chiarella and He (2003)
- HAM with market maker and noise traders, suitable to test empirically
- Evidence of both groups
- Evidence of switching and added value market maker
- Replicating stylized facts
- To do: forecasting?